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NONLINEAR FILTRATION IN CRACKED POROUS MATERIALS

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Stationary filtration to a well and to a gallery in a cracked porous medium is investigated with the strong dependence of structural-mechanical properties of the medium on the pressure of the filtering fluid being taken into account.

Motion in cracked-porous materials is usually described on the basis of a continual model within whose framework the material is considered as two coexistent fictitious porous media; filtration therein corresponds to independent progress over the porous modules and over the system of cracks in the presence of mutual fluid transfer [1]. The equations for the unknown mean pressures in the cracks and in the modules are obtained in [2]; they are analogous in structure to the heat-conduction equations in a heterogeneous medium (see [3-5], for example).

However, in contrast to the majority of heat-conduction processes, a strong nonlinear dependence of the cracked porosity and permeability (analogous to the specific heat and the heat conduction in the corresponding thermal problem) on the pressure within the cracks is characteristic for filtration processes in cracked and cracked porous media. This pressure dependence results in the appearance of a number of qualitatively new effects (finiteness of the

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pressure pulse propagation rate, origination of zones with fully closed cracks, etc.), which influence the observed characteristic of the filtration processes quite strongly. Effects of such kind were apparently first examined in [6] in application to the problem of strong thermal wave propagation due to nonlinear radiant heat conduction in the initial stage of an explosion in a gas, and in [7] in application to problems of filtering a compressible gas in a porous material with invariant properties.

In the case under consideration, in contrast to [7], the nonlinearity is not associated with the compressibility of the filtering gas but with the dependence of the structural—mechanical properties of the material on its state of stress and the pressure in the cracks. Moreover, filtration is performed not by a single macroscopic homogeneous medium but by modules and by cracks simultaneously. The presence of transfer between modules and cracks by the fluid results in a certain modification of the effects mentioned as compared with their appearances in "single-phase" processes. These processes are considered below only in an example of stationary problems on filtration to a single perfect well or to a gallery.

The system of equations describing stationary filtration in domains where cracks are open has the form [2]

$$\gamma\nabla\left[\left(\frac{p_1-\sigma}{p^0-\sigma}\right)^3\nabla p_1\right]+\frac{p_2-p_1}{\tau}=0,\quad \varkappa\Delta p_2-\frac{p_2-p_1}{\tau}=0. \tag{1}$$

The expressions for the effective piezoconductivity coefficients γ and $\varkappa$ and the relaxation time τ of the process of transfer by a fluid between the cracks and modules are given in [2]. The crack permeability is described in (1) by using a scalar function, which corresponds to filtration in a material of isotropic structure in a state of multilateral compression. These same equations describe plane filtration motion also in a material all of whose cracks are oriented in the plane of the motion (as is quite usual for sedimentary strata). In this case σ is understood to be the compressive stress normal to this plane.

In the domain where the cracks are closed, there is filtration only in the modules, describable by the standard equation

$$\varkappa\Delta p_2=0. \tag{2}$$

Let us note that the representation about the existence of two filtration zones in cracked-porous collectors was indeed introduced earlier (see [8], for example), but equations were used here that are, in principle, different from those obtained in [2]. Besides the usual boundary conditions on the boundaries of the flow domain, conditions on the equality of the pressure p_1, the critical magnitude σ, and the continuity of the pressure p_2 and the normal component of the flow in the modules, as well as the condition that the flow in the cracks vanish on the surface separating the domains with open and closed cracks are imposed on the solution of (1) and (2).

For the axisymmetric problem of filtration to a perfect well we have boundary conditions in the form

$$\begin{gathered}p_1=p_2^+=p^0,\ r=R;\ p_2^-=p_0,\ r=r_0;\\ \lim_{r\to r_*}\left(\frac{p_1-\sigma}{p^0-\sigma}\right)^3\frac{dp_1}{dr}=0,\ p_2^+=p_2^-,\ \frac{dp_2^+}{dr}=\frac{dp_2^-}{dr},\ p_1=\sigma,\ r=r_*.\end{gathered} \tag{3}$$

(for definiteness, the ordinary conditions of a pressure mode on the outer contour r = R and the condition of equality of p_2 to the cutter pressure on the well contour are taken; it is possible to give other conditions also). The boundary conditions for the problem of filtration to a gallery have the same form but with the replacement of r by x and r_0, r_*, and R by 0, x_*, and X.

Let us introduce the dimensionless characteristics

$$\eta=\frac{r}{R},\ \{\varphi_1,\ \varphi_2\}=\left\{\frac{p_1,\ p_2}{p^0-\sigma}\right\},\ \{\mu_\sigma,\ \mu_0,\ \mu^0\}=\left\{\frac{\sigma,\ p_0,\ p^0}{p^0-\sigma}\right\}.$$

Equations (1) are written in the form

$$\Delta(\varphi_1-\mu_\sigma)^4=4\lambda^2(\varphi_1-\varphi_2),\ \varepsilon\Delta\varphi_2=\lambda^2(\varphi_2-\varphi_1),\ \eta_*\leqslant\eta\leqslant 1, \tag{4}$$

where

$$\lambda^2 = \frac{R^2}{\tau\gamma}, \quad \varepsilon = \frac{\varkappa}{\gamma} \ll 1. \tag{5}$$

In the case of an influx to the gallery, system (4) has the first integral

$$\varphi_1 = \mu_\sigma + (A\eta + B - 4\varepsilon\varphi_2)^{1/4}, \tag{6}$$

and the function φ_2 satisfies the equation

$$\varepsilon d^2\varphi_2/d\eta^2 - \lambda^2\varphi_2 = -\lambda^2[\mu_\sigma + (A\eta + B - 4\varepsilon\varphi_2)^{1/4}], \quad \eta_* \leqslant \eta \leqslant 1. \tag{7}$$

For a well we obtain, respectively,

$$\varphi_1 = \mu_\sigma + (A\ln\eta + B - 4\varepsilon\varphi_2)^{1/4}, \tag{8}$$

$$\varepsilon\left(\frac{1}{\eta}\frac{d}{d\eta} + \frac{d^2}{d\eta^2}\right)\varphi_2 - \lambda^2\varphi_2 = -\lambda^2[\mu_\sigma + (A\ln\eta + B - 4\varepsilon\varphi_2)^{1/4}], \quad \eta^* \leqslant \eta \leqslant 1. \tag{9}$$

The solution of (7) and (9) can be constructed by the method of mergeable asymptotic expansions [9]. In this case a boundary layer exists near the boundary $\eta = \eta_*$. Using the boundary conditions (3) we first obtain certain estimates.

Taking (2) and (6) into account, conditions (3) for a gallery take the form

$$\begin{gathered} A + B - 4\varepsilon\varphi_2(1) = 1, \quad A\eta_* + B - 4\varepsilon\varphi_2(\eta_*) = 0, \\ A - 4\varepsilon(d\varphi_2/d\eta)_* = 0, \quad (d\varphi_2/d\eta)_*\eta_* + \mu_0 = \varphi_2(\eta_*), \end{gathered} \tag{10}$$

where the asterisk at the derivative denotes that it is evaluated for $\eta = \eta_*$.

Two more constants are added to the arbitrary constants A, B when integrating (7). The four constants and the unknown location of the boundary $\eta = \eta_*$ of the crack closure are determined from (10) and the condition for merger of the expansions.

From (10) we obtain the equality

$$4\varepsilon[\varphi_2(\eta_*) - \varphi_2(1) - (d\varphi_2/d\eta)_*(\eta_* - 1)] = 1. \tag{11}$$

The pressure drop in the modules $\varphi_2(\eta_*) - \varphi_2(1)$ depends substantially on the closeness of the critical value of σ to p^0. For $\sigma \to p^0, \varphi_2(\eta_*) - \varphi_2(1) = o(\varepsilon)$ the boundary is $\eta_* \to 1$; therefore the cracks are closed in the whole domain. For a cracked-porous collector with $\varepsilon \ll 1$ it can be assumed that the mentioned pressure drop will not be too large; consequently, from (11),

$$(d\varphi_2/d\eta)_* = o(\varepsilon^{-1}). \tag{12}$$

We then find from (10)

$$A = o(1), \quad B = o(\varepsilon), \quad \eta_* = o(\varepsilon). \tag{13}$$

Conditions (3) for a well have the form

$$\begin{gathered} B - 4\varepsilon\varphi_2(1) = 1, \quad A\ln\eta_* + B - 4\varepsilon\varphi_2(\eta_*) = 0, \\ A - 4\varepsilon(d\varphi_2/d\eta)_*\eta_* = 0, \quad (d\varphi_2/d\eta)_*\eta_*\ln\frac{\eta_*}{\eta_0} + \mu_0 = \varphi_2(\eta_*), \end{gathered} \tag{14}$$

from which the estimates (12) and

$$A = o(\eta_*), \quad B = o(1), \quad \eta_* = o(\eta_0) \tag{15}$$

follow. Therefore, in the case under consideration, the boundary of crack closure is sufficiently close to a gallery (well).

Let us proceed to construct the solution of (7) and (10). Taking account of (13), we set $B = B_0\varepsilon$ in (7) and (10); then $A = 1$ from the first condition in (10). The zeroth approximation of the external expansion has the form $\varphi_2 = \mu_\sigma + \eta^{1/4}$.

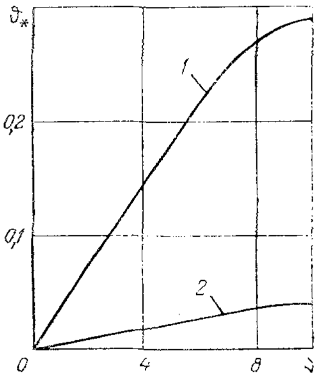

Fig. 1

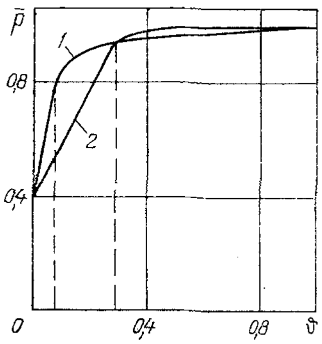

Fig. 2

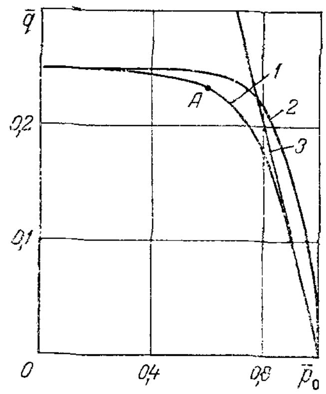

Fig. 3

Fig. 1. Influence of the pressure factor ν on the location of the crack closure front: 1) $\varepsilon = 10^{-2}$; 2) 10^{-3}.

Fig. 2. Pressure distribution in a collector: 1) $\vartheta_* = 0.07$; 2) $\vartheta_* = 0.29$; $\varepsilon = 10^{-2}$.

Fig. 3. Dependence of the debit on the face pressure: 1) $\bar{\sigma} = 0.2$; 2) 0.5; 3) porous collector.

Setting $\eta = \sqrt{\varepsilon}\xi$ in (7), we obtain the equation

$$d^2\varphi_2/d\xi^2 - \lambda^2\varphi_2 = -\lambda^2[\mu_\sigma + (A\sqrt{\xi} + B_0\varepsilon - 4\varepsilon\varphi_2)^{1/4}];$$

hence the zeroth approximation of the internal expansion is

$$\varphi_2 = \mu_\sigma + C_1 \exp[-\lambda(\xi - \xi_*)] + C_2 \exp[\lambda(\xi - \xi_*)].$$

From the merger condition $C_2 = 0$, and from the last equation in (10),

$$C_1 = (\mu_0 - \mu_\sigma)\left(1 + \frac{\lambda\eta_*}{\sqrt{\varepsilon}}\right)^{-1}.$$

The composite expansion has the form

$$\varphi_2 = \mu_\sigma + \eta^{1/4} + (\mu_0 - \mu_\sigma)\left(1 + \frac{\lambda\eta_*}{\sqrt{\varepsilon}}\right)^{-1}\exp\left[-\frac{\lambda(\eta - \eta_*)}{\sqrt{\varepsilon}}\right], \tag{16}$$
$$\eta_* \leqslant \eta \leqslant 1.$$

Taking (16) into account, we obtain an equation for the coordinate $\xi = \xi_*$ of the crack closure boundary from the third condition in (10)

$$1 = \varepsilon^{5/8}\xi_*^{-3/4} - 4\sqrt{\varepsilon}(\mu_0 - \mu_\sigma)\left(\frac{1}{\lambda} + \xi_*\right)^{-1}. \tag{17}$$

Let us estimate the magnitude of the parameter λ. In conformity with the examples in [8], we take $\gamma = 0.6\ m^2/sec$, $\tau = 180$ sec for the cracked-porous collector. For $R = 100$ m (half the distance between wells), $\lambda^2 \approx 10^2$.

In case $\lambda \gg 1$ we obtain from (17)

$$\xi_* = 4\sqrt{\varepsilon}(\mu_\sigma - \mu_0), \tag{18}$$

which is in agreement with (13).

It follows from (6) and (16) that $\varphi_1 < \varphi_2$ near $\eta = 1$ (the overflow from the modules to crack) and $\varphi_1 > \varphi_2$ near $\eta = \eta_*$ (reverse overflow). The expression for the derivative

$$(d\varphi_1/d\eta)_* = \frac{1}{4}\varepsilon^{-3/8}\xi_*^{-3/4}$$

characterizes the "blurring" of the crack closure front for $\varepsilon \neq 0$.

In the case of filtration to a well [Eqs. (9) and (14)], we just select the internal variable. According to the second condition in (14), the internal variable is determined by the equation $\ln \eta = f(\varepsilon, \xi)$, $f(0, \xi) \neq 0$, where $f(\varepsilon, \xi)$ is a series in certain positive powers of ε. Hence, the differentiation operator in (9) is

$$\frac{1}{\eta}\frac{d}{d\eta} + \frac{d^2}{d\eta^2} = (f' \exp f)^{-2}\left(\frac{d^2}{d\xi^2} - \frac{f''}{f'}\frac{d}{d\xi}\right), \tag{19}$$

where the primes denote the derivatives with respect to ξ. Expanding the operator (19) in a power series in ε, we determine the first terms in the expansion $f(\varepsilon, \xi)$ from the condition of simplicity of integrating the transformed equation (9). In a zeroth approximation the coefficients in (9) should be constant. We obtain

$$\eta = \exp[\beta_1 + \beta_2 \sqrt{\varepsilon}\,\xi],\ \beta_1 \neq 0,\ \beta_2 \neq 0. \tag{20}$$

In the zeroth approximation (9) has the form

$$d^2\varphi_2/d\xi^2 - \omega^2\varphi_2 = -\omega^2[\mu_\sigma + (A\beta_1 + 1)^{1/4}],\ \omega = \lambda\beta_2 \exp\beta_1.$$

Filtration to a well is described by the composite expansion

$$\begin{gathered}\varphi_2 = \mu_\sigma + \left(1 - \frac{\ln\eta}{\beta_1}\right)^{1/4} + (\mu_0 - \mu_\sigma)\left(1 + b\ln\frac{\eta_*}{\eta_0}\right)^{-1}\left(\frac{\eta}{\eta_*}\right)^{-b},\\ b = \lambda\varepsilon^{-1/2}\exp\beta_1,\ \eta_* \leqslant \eta \leqslant 1.\end{gathered} \tag{21}$$

The front of crack closure is determined for $\lambda >> 1$ by the equation

$$\ln\eta_* = \ln\eta_0 - 4\,\varepsilon\beta_1(\mu_\sigma - \mu_0). \tag{22}$$

The solution of (21) and (22) turns out to be dependent on the parameter β_1 whereupon only terms of zeroth order in ε will be retained upon compliance with the substitution of (20) into (9). It can be shown that the dependence on β_1 is weak in (21) and (22).

Let us consider a simplified model of fluid filtration to a well by neglecting the permeability of the modules for $\eta > \eta_*$. As $\varepsilon \to 0$ it follows from (8), (9), and (2)

$$\begin{gathered}\varphi_1 = \varphi_2^+ = \mu_\sigma + (A\ln\eta + B)^{1/4},\ \eta_* \leqslant \eta \leqslant 1,\\ \varphi_2^- = \alpha\ln\eta + \beta,\ \eta_0 \leqslant \eta \leqslant \eta_*.\end{gathered}$$

Determining the constants from the boundary conditions

$$\begin{gathered}\varphi_1 = \varphi_2^+ = \mu^0,\ \eta = 1;\ \varphi_2^- = \mu_0,\ \eta = \eta_0;\\ \varphi_1 = \varphi_2^+ = \varphi_2^- = \mu_\sigma,\ \eta = \eta_*,\end{gathered}$$

we find

$$\begin{gathered}\varphi_1 = \varphi_2^+ = \mu_\sigma + (1 - \ln\eta/\ln\eta_*)^{1/4},\ \eta_* \leqslant \eta \leqslant 1,\\ \varphi_2^- = \mu_0 + (\mu_\sigma - \mu_0)[\ln(\eta_*/\eta_0)]^{-1}\ln\frac{\eta}{\eta_0},\ \eta_0 \leqslant \eta \leqslant \eta_*.\end{gathered} \tag{23}$$

We obtain the boundary of crack closure from the equality of the flows for $\eta = \eta_*$

$$\ln\eta_* = \ln\eta_0[1 + 4\varepsilon(\mu_\sigma - \mu_0)]^{-1}, \tag{24}$$

which corresponds to (22).

For a gallery we obtain, respectively,

$$\begin{gathered}\varphi_1 = \varphi_2^+ = \mu_\sigma + [(\eta - \eta_*)/(1 - \eta_*)]^{1/4},\ \varphi_2^- = \mu_0 + (\mu_\sigma - \mu_0)\,\eta/\eta_*,\\ \eta_* = 4\varepsilon(\mu_\sigma - \mu_0)[1 + 4\varepsilon(\mu_\sigma - \mu_0)]^{-1} \approx 4\varepsilon(\mu_\sigma - \mu_0)\end{gathered}$$

[the last equation agrees with (18)].

The dependence (24) for the crack closure front location on the pressure factor $\nu = \mu_\sigma - \mu_0$ is shown in Fig. 1, where the coordinate $\vartheta = 1 - \ln \eta / \ln \eta_0$ is used. For sufficiently small ε the pressure factor influences the boundary location weakly.

The pressure distribution (23) in the collector is shown in Fig. 2 for two boundary locations. The dimensionless pressure $\bar{p} = p/p_0$ is plotted along the vertical axis. The zone is characterized by a large pressure gradient in the modules [corresponds to (12)]. For the pressure in the collector changes more slowly and depends weakly on the position of the boundary $\vartheta = \vartheta_*$.

For $p_0 > \sigma$ the cracks are open in the whole domain. For the conditions $\varphi_1 = \varphi_2 = \mu_0$ at $\eta = \eta_0$ we obtain

$$\varphi_1 = \varphi_2 = \mu_\sigma + [(\nu^4 - 1) \ln \eta / \ln \eta_0 + 1]^{1/4}, \quad \eta_0 \leqslant \eta \leqslant 1. \tag{25}$$

Let us clarify the dependence of the debit on the face pressure $\bar{p}_0$. Taking account of (23)-(25), the well debit is determined to the accuracy of a constant by the expressions

$$q = \gamma(\nu^4 - 1)(4\eta_0 \ln \eta_0)^{-1}, \quad \bar{p}_0 > \bar{\sigma}.$$
$$q = -\gamma / 4\eta_0 \ln \eta_0, \quad \bar{p}_0 < \bar{\sigma}. \tag{26}$$

The dependences (26) are shown in Fig. 3 for two values of the critical pressure $\bar{\sigma}$. The quantity $\bar{q} = \eta_0 \ln \eta_0 q \gamma^{-1}$ is plotted along the vertical axis. The effect of crack closure strongly distorts the display diagram as compared with a porous collector (the diagram for a porous collector with permeability factor γ is displayed by the line 3). The flow in a cracked-porous collector with closed cracks can be considerably less than the flow in a porous collector.

As follows from (26), $\bar{q} = {}^1/_4$ for $\bar{p}_0 < \bar{\sigma}$. The asymptotic behavior of the display curve is sufficiently characteristic for cracked-porous collectors (see [8] for example). Let us note that this phenomenon is usually associated with the degassing of petroleum.

The possibility of crack closure in a collector was noted in [10]. Asymptotic methods in filtration problems were used in [11, 12].

The effect of crack closure in an elastically compressible cracked-porous collector should be taken into account in estimating petroleum reserves. We assume that the debit $\bar{q} = 0.25$ is determined for a face pressure of $\bar{p}_0 = 0.6$ (the point A in Fig. 3) for a preliminary estimate of the promise of a deposit in a well; hence the well productivity factor is $K = K_1 = 0.6$. Meanwhile, as follows from Fig. 3, for open cracks $K = K_2 \leq 0.8$. The productivity is in agreement with the permeability k in the dimensionless writing used. High porosity m usually corresponds to high permeability. For instance, when using the power-law dependence $k = m^a$ [13] for a fixed medium, we obtain $m_1 = 0.1 m_2$. Therefore, the petroleum reserves will be substantially reduced.

Therefore, it follows from the analysis presented above that the dependence of the structural-mechanical characteristics of cracked-porous materials on the fluid pressure actually results in substantially nonlinear behavior of the filtration fluid, even if the fluid is Newtonian. Neglecting this circumstance can cause significant errors in determining the effective properties of a material by the display curve and, consequently, an error in estimating the real petroleum reserved in cracked-porous collectors.

NOTATION

p, fluid pressure; γ, $\varkappa$, effective piezoconductivity factors; τ, relaxation time; σ, compressive stress; r, x, coordinates; R, X, values of corresponding coordinates on the supply contour; η, ϑ, dimensionless coordinates; φ, $\bar{p}$, dimensionless pressures; μ, boundary value of the dimensionless pressure φ; λ, ε, dimensionless parameters in (5); A, B, constant factors in (6); ξ, dimensionless internal variable; $f(\varepsilon, \xi)$, function in (20), β_1, β_2, constants in (20); b, parameter in (21); ν, pressure factor in (25); q, dimensionless debit; K, productivity factor; k, permeability; m, porosity, a, a parameter.

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